



PON VIDYASHRAM GROUP OF CBSE SCHOOLS

VACATION HOME TEST (2017-2018)

STD XI

MATHEMATICS

Marks: 100

General Instructions :

1. All questions are compulsory.
2. The question paper consists of 26 questions divided into 3 sections A, B and C. Section A comprises of 6 questions of 1 mark each, Section B comprises of 13 questions of 4 marks each and Section C comprises of 7 questions of 6 marks each.
3. There is no overall choice. However internal choice has been provided in 4 questions of 4 marks each and 2 questions of 6 marks each. You have to attempt only one of the alternatives in all such questions.
4. Write the serial number of question before attempting.
5. Use of calculator is not permitted. However, you may ask for logarithmic and statistical tables, if required.

SECTION-A

Question number 1 to 6 carry 1 mark each.

1. If  $\sin x = \frac{3}{5}$ , then find  $\cos 2x$ .
2. Find the multiplicative inverse of  $(4 - 3i)$ .
3. What is the domain of the real valued function  $f(x) = \frac{1}{3x-2}$  ?
4. Find the value of  $\lim_{x \rightarrow 1} \frac{x^{\frac{1}{5}} - 1}{x^{\frac{1}{6}} - 1}$ .
5. Line through the points  $(-2, 6)$  and  $(4, 8)$  is perpendicular to the line through the points  $(8, 12)$  and  $(x, 24)$ . Find the value of  $x$ .
6. Find the component statement of "0 is a positive integer or a negative integer" ?

SECTION-B

Question number 7 to 19 carry 4 marks each.

7. Let  $U = \{1, 2, 3, 4, 5, 6, 8\}$ ,  $A = \{2, 3, 4\}$  and  $B = \{3, 4, 5\}$ . Show that :  
 $(A \cup B)' = A' \cap B'$  and  $(A \cap B)' = A' \cup B'$ .

8. Find the domain and range of the real function  $f(x) = \sqrt{9-x^2}$ .
9. Find the general solution of the following equation :  
 $\sin x + \sin 3x + \sin 5x = 0$ .
10. In any  $\Delta ABC$ , prove that  $\frac{a^2+b^2}{a^2+c^2} = \frac{1+\cos(A-B)\cos C}{1+\cos(A-C)\cos B}$ .

OR

Prove that :  $(\cos x + \cos y)^2 + (\sin x - \sin y)^2 = 4 \cos^2 \left( \frac{x+y}{2} \right)$

11. A committee of two persons is selected from two men and two women. What is the probability that the committee will have (i) no man ? (ii) one man ?
12. Prove that :  $\cos A \cdot \cos 2A \cdot \cos 2^2 A \cdot \cos 2^3 A \cdot \dots \cos 2^{n-1} A = \frac{\sin 2^n A}{2^n \sin A}$ .
13. Find the sum to  $n$  terms of the sequence 8, 88, 888, 8888, .....

OR

If  $a, b, c$  and  $d$  are in G.P., prove that

$$(a^n + b^n), (b^n + c^n), (c^n + d^n) \text{ are in G.P.}$$

14. A G.P. consists of an even number of terms. If the sum of all the terms is 5 times the sum of terms occupying odd places, then find the common ratio.
15. Evaluate :  $\lim_{x \rightarrow 0} \frac{\tan x - \sin x}{x^3}$

OR

Find the derivative of  $x \sin x$  with respect to  $x$  by first principle.

16. Convert the complex number  $z = \frac{-16}{1+i\sqrt{3}}$  into polar form.
17. How many 5-digit telephone number can be formed using the digits 0-9 if each number starts with 67 and no digit appears more than once ?
18. Using principle of mathematical induction. Prove that :

$$1^2 + 3^2 + 5^2 + \dots + (2n-1)^2 = \frac{n(2n-1)(2n+1)}{3}, n \in N.$$

OR

Using principle of mathematical induction, prove that  $41^n - 14^n$  is multiple of 27.

19. Show that :  $\frac{(\sin 7x + \sin 5x) + (\sin 9x + \sin 3x)}{(\cos 7x + \cos 5x) + (\cos 9x + \cos 3x)} = \tan 6x$ .

## SECTION-C

Question number 20 to 26 carry 6 marks each.

20. A firm which involves in the production of tea and coffee of a particular kind, conduct a survey on the group of 600 people, they found that 150 people like tea and 225 like coffee of that kind and 100 people like both the tea and coffee, and remaining people like the other usual drinks. Find how many people neither like tea nor coffee of that kind ? What was the motive/action behind this survey ?
21. If the arcs of the same lengths in two circles subtend angles  $65^\circ$  and  $110^\circ$  at the centre, find the ratio of their radii.

22. Solve the following system of inequalities graphically :

$$3x + 2y \leq 150, x + 4y \leq 80, x \leq 15, x \geq 0, y \geq 0.$$

23. Find the term independent of  $x$  in the expansion of  $\left(2x - \frac{1}{x}\right)^{10}$ ,  $x \neq 0$ .

24. A line is such that its segment between the lines  $5x - y + 4 = 0$  and  $3x + 4y - 4 = 0$  is bisected at the point  $(1, 5)$ . Obtain its equation.

25. Find the sum of the following series upto  $n$  terms :

$$\frac{1^3}{1} + \frac{1^3 + 2^3}{1+3} + \frac{1^3 + 2^3 + 3^3}{1+3+5} + \dots$$

OR

Show that :  $\frac{1 \times 2^2 + 2 \times 3^2 + \dots + n \times (n+1)^2}{1^2 \times 2 + 2^2 \times 3 + \dots + n^2 \times (n+1)} = \frac{3n+5}{3n+1}$ .

26. The diameters of circles (in mm) drawn in a design are given below :

Diameters	33 - 36	37 - 40	41 - 44	45 - 48	49 - 52
Number of circles	15	17	21	22	25

Calculate the standard deviation and mean diameter of the circles.

OR

Given that  $\bar{x}$  is the mean and  $\sigma^2$  is the variance of  $n$  observations  $x_1, x_2, x_3, \dots, x_n$ . Prove that the mean and variance of the observations  $ax_1, ax_2, ax_3, \dots, ax_n$  are  $a\bar{x}$  and  $a^2\sigma^2$  respectively ( $a \neq 0$ ). What can we further conclude from the above interpretation?

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OMIT the Marked question

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