



PON VIDYASHRAM
GROUP OF CBSE SCHOOLS
VACATION HOME TEST- (2017- 2018)
CLASS – XII

TIME: 3 hrs
MARKS: 100

MATHEMATICS

General Instructions:

- (i) All questions are compulsory.
- (ii) Questions 1–4 in section A are very short answer type questions carrying 1 mark each.
- (iii) Questions 5–12 in section B are short answer type questions carrying 2 marks each.
- (iv) Questions 13–23 in section C are long answer I type questions carrying 4 marks each.
- (v) Questions 24–29 in section D are long answer II type questions carrying 6 marks each.

SECTION A

1. If A is a skew symmetric matrix of order 3×3 then prove that $|A| = 0$ 1
2. Check the continuity of the function $f(x) = \begin{cases} x & \text{if } x \leq 1 \\ 5 & \text{if } x > 1 \end{cases}$ at $x = 1$. 1
3. Find the antiderivative of $\sin 2x$ by the method of inspection 1
4. If the line $\frac{x-1}{2} = \frac{y+4}{1} = \frac{z-7}{2}$ is parallel to the plane $3x - 2y + cz = 14$ then find the value of c . 1

SECTION B

5. Determine the value of x for which the matrix $A = \begin{pmatrix} x+1 & -3 & 4 \\ -5 & x+2 & 2 \\ 4 & 1 & x-6 \end{pmatrix}$ is singular. 2
6. Differentiate $\tan^{-1} \sqrt{\frac{1-\cos x}{1+\cos x}}$, $-\frac{\pi}{4} < x < \frac{\pi}{4}$ w.r.t. x . 2
7. Verify Mean Value Theorem for $f(x) = (x-1)(x-2)(x-3)$ in $[1, 4]$ 2
8. Find the approximate value of $f(3.01)$ when $f(x) = 3x^2 + 5x + 4$. 2
9. Find the angle between the pair of lines $\vec{r} = 2\hat{i} - 5\hat{j} + \hat{k} + \lambda(3\hat{i} + 2\hat{j} + 6\hat{k})$ and $\vec{r} = (7 + \mu)\hat{i} + (-6 + 2\mu)\hat{j} + 2\mu\hat{k}$ 2
10. Evaluate $P(A \cup B)$ if $2P(A) = P(B) = \frac{5}{13}$ and $P(A|B) = \frac{2}{5}$ 2
11. Reshma wishes to mix two types of food P and Q in such a way that the vitamin contents of the mixture contain atleast 8 units of vitamin A and 11 units of vitamin B. Food P costs ₹ 60/kg

and food Q costs ₹ 80/kg. Food P contains 3 units/kg of vitamin A and 5 units/kg of vitamin B while food Q contains 4 units/kg of vitamin A and 2 units/kg of vitamin B. Formulate the L.P.P for minimum cost of the mixture. 2

12. Evaluate $\int_0^2 (x - [x]) dx$ 2

SECTION C

13. Find the real solutions of the equation 4

$$\tan^{-1} \sqrt{x(x+1)} + \sin^{-1}(\sqrt{x^2+x+1}) = \frac{\pi}{2}$$

OR

Prove that $\tan^{-1} \left(\frac{\sqrt{1+\cos x} + \sqrt{1-\cos x}}{\sqrt{1+\cos x} - \sqrt{1-\cos x}} \right) = \frac{\pi}{4} - \frac{x}{2}$ where $\pi < x < \frac{3\pi}{2}$

14. Using properties of determinants prove that 4

$$\begin{vmatrix} a^2 & bc & ac+c^2 \\ a^2+ab & b^2 & ac \\ ab & b^2+bc & c^2 \end{vmatrix} = 4a^2b^2c^2$$

OR

Using properties of determinants prove that

$$\begin{vmatrix} x & x^2 & 1+px^3 \\ y & y^2 & 1+py^3 \\ z & z^2 & 1+pz^3 \end{vmatrix} = (1+pxyz)(x-y)(y-z)(z-x)$$

15. If $x^2 + y^2 = a - \frac{1}{a}$ and $x^4 + y^4 = a^2 + \frac{1}{a^2}$ then prove that $\frac{dy}{dx} = \frac{1}{x^3y}$ 4

16. Evaluate $\int \frac{dx}{4\cos^3 x - 3\cos x}$ 4

OR

Evaluate $\int x^2 \log(1+x) dx$

17. Evaluate $\int_0^1 \cot^{-1}(1-x+x^2) dx$ 4

18. Evaluate $\int \frac{dx}{\sqrt{5x^2-2x}}$ 4

19. If \vec{a} and \vec{b} are two vectors such that $|\vec{a} + \vec{b}| = |\vec{a}|$ then prove that $2\vec{a} + \vec{b}$ is perpendicular to \vec{b} . 4

20. If \hat{a} and \hat{b} are unit vectors and θ is the angle between them then prove that $\cos \theta/2 = \frac{1}{2} |\hat{a} + \hat{b}|$. 4

21. 40% students of a college reside in hostel and the remaining reside outside. At the end of the year, 50% of the hostellers got A grade while from outside students, only 30% got A grade in

- the examination. At the end of year, a student of the college was chosen at random and was found to get A grade. What is the probability that the selected student was a hosteller? 4
22. Three distinguishable balls are distributed in three cells. Find the conditional probability that all the three occupy the same cell given that atleast two of them are in the same cell. Which values are reflected from tricolour of our national flag? 4
23. Solve the following L.P.P. graphically. 4
- Maximise $Z = 8000x + 12000y$
 Subject to constraints are $9x + 12y \leq 180$
 $x + 3y \leq 30$
 $x, y \geq 0$

SECTION D

24. Let $A = \{1, 2, 3, \dots, 9\}$ and R be the relation in $A \times A$ defined by $(a, b) R (c, d)$. If $a + d = b + c$ for $a, b, c, d \in A$, prove that R is an equivalence relation. Also obtain the equivalence class $(2, 5)$. 6

OR

Let $A = \mathbb{R} - \{3\}$ and $B = \mathbb{R} - \{1\}$. Let $f: A \rightarrow B$ defined by $f(x) = \frac{x-2}{x-3}$ for all $x \in A$. Then show that f is bijective. Hence find $f^{-1}(x)$. Also find the value of $f^{-1}(17)$.

25. Find the area of the region bounded by the curves $y = \tan x$, the tangent drawn to the curve $y = \tan x$ at $x = \frac{\pi}{4}$ and the x -axis using integration. 6

OR

Find the area enclosed by the curves $y = |x - 1|$ and $y = 1 - |x - 1|$ using integration.

26. Show that the four points $(0, -1, -1)$, $(-4, 4, 4)$, $(4, 5, 1)$ and $(3, 9, 4)$ are coplanar. Find the equation of the plane containing them. 6

OR

Find the angle between the tangents to the parabolas $y^2 = 4ax$ and $x^2 = 4by$ at their point of intersection other than the origin.

27. Find the particular solution of the differential equation $xe^{y/x} - y \sin\left(\frac{y}{x}\right) + \frac{xdy}{dx} \sin\left(\frac{y}{x}\right) = 0$ given that $y = 0$ when $x = 1$. 6

28. If $A = \begin{pmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{pmatrix}$, find A^{-1} . Using A^{-1} solve the system of equations 6

$$2x - 3y + 5z = 11$$

$$3x + 2y - 4z = -5$$

$$x + y - 2z = -3$$

29. A plot is in the form of a rectangle with a semicircular plot along one of the shorter sides. The perimeter of the total plot is 100 m. A farmer wants to use the maximum area for growing the grass. Find the dimensions of the plot so that maximum grass can be planted. 6